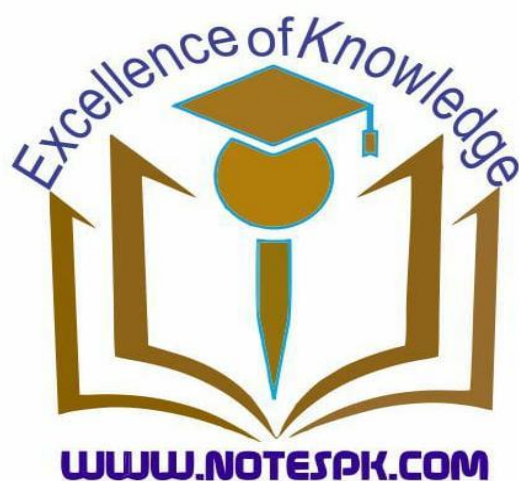
MATHEMATICS 9th Science Group

7/18/2020

Chapter 2.

REAL AND COMPLEX NUMBERS

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Rational number:

A number which can be written in the form of $\frac{p}{q}$ $, where\ p, q\ \epsilon\ Z\ \Lambda \neq 0$ is called a rational number.

e.g. $\frac{3}{4}, \frac{22}{7}, \frac{2}{6}$.

Irrational number:

A real number which cannot be written in the form of $\frac{p}{q}$ $, where\ p, q\ \epsilon\ Z\ \Lambda \neq 0$ is called an irrational number.

e.g. $\sqrt{2}$ $,\sqrt{5}$

Real number:

The field of all rational and irrational numbers is called the real numbers, or simply the "reals," and denoted $\mathbb{R}$.

Terminating decimal:

A decimal which has only a finite number of digits in its decimal part, is called terminating decimal. e.g. 202.04, 0.25, 0.5 example of terminating decimal.

Recurring decimal:

A decimal in which one or more digits repeats indefinitely is called recurring decimal or periodic decimal.

e.g. 0.33333 , 21.134134 … …

Exercise 2.1

Question.1. Identify which of the following are rational and irrational numbers

(i). $\sqrt{3}$

Solution.

Is an irrational number.

(ii). $\frac{1}{6}$

Solution.

Is a rational number.

(iii). π

Solution.

Is an irrational number.

(iv). $\frac{15}{7}$

Solution.

Is a rational number.

(v). 7.25

Solution.

Is a rational number.

(vi). $\sqrt{29}$

Solution.

Is an irrational number.

Question.2. Convert the following fractions into decimal fraction.

(i) $\frac{17}{25}$

Solution.

$$\mathbf{0.68}$$

(ii) $\frac{19}{4}$

Solution.

$$\mathbf{4.75}$$

(iii) $\frac{57}{8}$

Solution.

$$\mathbf{7.125}$$

(iv) $\frac{205}{18}$

Solution.

$$\mathbf{11.3889}$$

(v) $\frac{5}{8}$

Solution.

$$\mathbf{0.625}$$

(vi) $\frac{25}{38}$

Solution.

$$\mathbf{0.65789}$$

Question.3. Which of the following statements are true and which are false?

(i). $\frac{2}{3}$ ***is an irrational number.***

Solution.

False.

(ii). π ***is an irrational number.***

Solution.

True.

(iii). $\frac{1}{9}$ ***is a terminating fraction.***

Solution.

False.

(iv). $\frac{3}{4}$ ***is terminating fraction.***

Solution.

True.

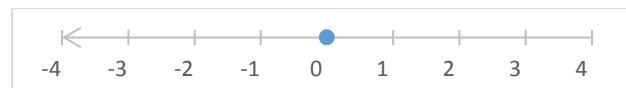
(v). $\frac{4}{5}$ ***is a recurring fraction..***

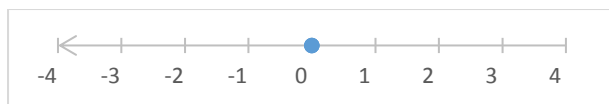
Solution.

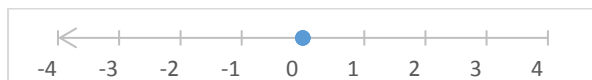
False.

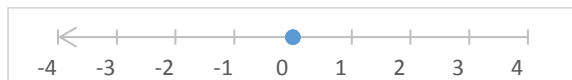
Question.4. Represent the following numbers on the number line

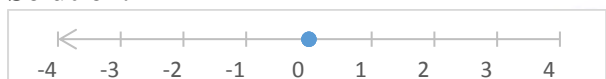
(i) $\frac{2}{3}$

Solution.

(ii). $-\frac{4}{5}$

Solution.

(iii). $1\frac{3}{4}$

Solution.

(iv). $-2\frac{5}{8}$

Solution.

(v). $\sqrt{5}$

Solution.

Question.5. Give a rational number between $\frac{3}{4}$ and $\frac{5}{9}$.

Solution.

The mean of the numbers is between given numbers. Therefore

$$required\ number\ is = \frac{\frac{3}{4}+\frac{5}{9}}{2}$$

$$= \frac{\frac{27+20}{36}}{2}$$

$$= \frac{\frac{47}{36}}{2}$$

$$= \frac{47}{36\times 2}$$

$$= \frac{47}{72}$$

Answer.

***Question*.6. Express the following recurring decimals as the rational number $\frac{p}{q}$, where p, q are integers and $q \neq 0$.**

(i). $0.\overline{5}$

Solution.

Let

$$x = 0.\overline{5}$$

That is

$$x = 0.5555\ldots \rightarrow (i)$$

Only one digit 5 is being repeated, multiply by 10 on both sides of (i), we have

$$10x = (0.5555\ldots)\times 10$$

$$10x = 5.5555\ldots \rightarrow (ii)$$

Subtracting (i) *from* (ii), *we have*

$$10x - x = 5.5555\ldots . -0.5555\ldots$$

$$9x = 5$$

$$x = \frac{5}{9}$$

$$0.\overline{5} = \frac{5}{9}$$

Answer.

(ii). $0.\overline{13}$

Solution.

Let

$$x = 0.\overline{13}$$

That is

$$x = 0.13131313\ldots \rightarrow (i)$$

Only two digits 13 is being repeated, multiply by 100 on both sides of (i), we have

$$100x = (0.13131313\ldots)\times 100$$

$$100x = 13.13131313\ldots \rightarrow (ii)$$

Subtracting (i) *from* (ii), *we have*

$$100x - x$$

$$= 13.13131313\ldots . -0.13131313\ldots$$

$$99x = 13$$

$$x = \frac{13}{99}$$

$$0.\overline{13} = \frac{13}{99}$$

Answer.

(iii). $0.\overline{67}$

Solution.

Let

$$x = 0.\overline{67}$$

That is

$$x = 0.67676767\ldots \rightarrow (i)$$

Only two digits 67 is being repeated, multiply by 100 on both sides of (i), we have

$$100x = (0.67676767\ldots)\times 100$$

$$100x = 67.67676767\ldots \rightarrow (ii)$$

Subtracting (i) *from* (ii), *we have*

$$100x - x$$

$$= 67.67676767\ldots . -0.67676767\ldots$$

$$99x = 67$$

$$x = \frac{67}{99}$$

$$0.\overline{67} = \frac{67}{99}$$

Answer.

Properties of Real Numbers:

Binary Operations:

A binary operation in a set A is a rule usually denoted by $*$ that assigns to any pair of elements of A to another element of A. e.g. two important binary operations are addition and multiplication in a set of real numbers.

Addition Laws:

Closure Law of Addition:

$$\forall\, a, b \in \Re \text{ then } a + b \in \Re \qquad \forall \text{ stands for all.}$$

Associative Law of Addition:

$$\forall\, a, b, c \in \Re \text{ then } a + (b + c) = (a + b) + c.$$

Additive Identity:

$$\forall\, a \in \Re, \exists\, 0 \in \Re \text{ such that } a + 0 = 0 + a = a.$$

$\exists$ *Stands for there exist and* 0 *is called the iditive identity.*

Additive Inverse:

$$\forall\, a \in \Re, \exists -a \in \Re \text{ such that } a + (-a) = -a + a = 0.$$

$-a$ *and* a *are called the idetive inverse of each other.*

Commutative Law for Addition:

$$\forall\, a, b \in \Re \text{ then } a + b = b + a.$$

Multiplication Laws:

Closure Law of Multiplication:

$$\forall\, a, b \in \Re \text{ then } ab \in \Re \qquad \forall \text{ stands for all.}$$

Associative Law of Multiplication:

$$\forall\, a, b, c \in \Re \text{ then } a(bc) = (ab)c.$$

Multiplicative Identity:

$$\forall\, a \in \Re, \exists\, 1 \in \Re \text{ such that } a.1 = 1.a = a.$$

$\exists$ *Stands for there exist and* 1 *is called the iditive identity.*

Multiplicative Inverse:

$$\forall\, a \in \Re, \exists\, a' = \frac{1}{a} \in \Re \text{ such that } a.\frac{1}{a} = \frac{1}{a}.a = 1.$$

a *and* $\frac{1}{a}$ *are called the idetive inverse of each other.*

Commutative Law for Multiplication:

$$\forall\, a, b \in \Re \text{ then } ab = ba.$$

Properties of Equality:

Reflexive property:

$$\forall\, a \in \Re \text{ then } a = a$$

Symmetric Property:

$$\forall\, a, b \in \Re \text{ and if } a = b \text{ then } b = a.$$

Transitive Property:

$$\forall\, a, b, c \in \Re, \text{ if } a = b \text{ and } b = c \text{ then } a = c.$$

Additive Property:

$$\forall\, a, b, c \in \Re, a = b \text{ then } a + c = b + c.$$

Multiplicative Property:

$$\forall\, a, b, c \in \Re, a = b \text{ then } ac = bc.$$

Cancellation Property w.r.t. addition:

$$\forall\, a, b, c \in \Re, a + c = b + c \text{ then } a = b.$$

Cancellation Property w.r.t. Multiplication:

$$\forall\, a, b, c \in \Re, ac = bc \text{ then } a = b.$$

Distributive property of multiplication over addition.

$$a(b + c) = ab + ac$$

Distributive property of multiplication over Subtraction.

$$a(b - c) = ab - ac$$

Properties of Inequalities (Order properties):

Trichotomy Property:

$$\forall\, a, b \in \Re$$

$$\text{either } a = b \text{ or } a > b \text{ or } a < b.$$

Transitive Property:

$$\forall\, a, b \in \Re$$

(i). $\text{if } a > b \text{ and } b > c \text{ then } a > c.$

(ii). $\text{if } a < b \text{ and } b < c \text{ then } a < c.$

Additive Property:

$$\forall\, a, b \in \Re$$

(i). $\text{if } a > b \text{ then } a + c > b + c.$

(ii). $\text{if } a < b \text{ and then } a + c < b + c.$

Multiplicative Properties:

$$\forall\, a, b, c \in \Re$$

$\text{If } c > 0$

(i). $if\ a > b\ then\ ac > bc.$
(ii). $if\ a < b\ and\ then\ ac < bc.$
$If\ c < 0$
(iii). $if\ a > b\ then\ ac < bc.$
(iv). $if\ a < b\ and\ then\ ac > bc.$

Exercise 2.2

Question.1. Identify the property used in the following.

(i). $\boldsymbol{a + b = b + a}$
Solution.
$Commutative\ property\ w.r.t\ Addition.$

(ii). $(\boldsymbol{ab})\boldsymbol{c} = \boldsymbol{a}(\boldsymbol{bc})$
Solution.
$Associative\ property\ w.r.t\ Multiplication.$

(iii). $\boldsymbol{7 \times 1 = 7}$
Solution.
$Multiplicative\ identity.$

(iv). $\boldsymbol{x > y\ or\ x = y\ or\ x < y}$
Solution.
$Trichotromy\ Property.$

(v). $\boldsymbol{ab = ba}$
Solution.
$Commutative\ property\ w.r.t\ Multiplication.$

(vi). $\boldsymbol{a + b = b + c \Rightarrow a = b}$
Solution.
$Cancellation\ Law\ w.r.t\ Addition.$

(vii). $\boldsymbol{5 + (-5) = 0}$
Solution.
$Additive\ Inverse.$

(viii). $\boldsymbol{7 \times \frac{1}{7}}$
Solution.
$Multiplicative\ Inverse.$

(ix). $\boldsymbol{a > b \Rightarrow\ ac > bc\ (c > 0)}$
Solution.
$Multplicative\ propert.$

Question.2. Fill in the following blanks by stating the properties of real numbers used.

$$\boldsymbol{3x + 3(y - x)}$$

Solution.
Given
$3x + 3(y - x) = 3x + 3y - 3x$
$Distributive\ property\ w.r.t\ muliplication\ over\ subtraction.$
$= 3x - 3x + 3y \quad commutative\ property\ w.r.t\ addition.$
$= 0 + 3y \quad additive\ inverse\ property.$
$= 3y \quad additive\ identity.$
Answer.

Question.3. Give the name of property used in the following.

(i). $\boldsymbol{\sqrt{24} + 0 = \sqrt{24}}$
Solution.
$Additive\ identity.$

(ii). $\boldsymbol{-\frac{2}{3}\left(5 + \frac{7}{2}\right) = \left(-\frac{2}{3}\right)(5) + \left(-\frac{2}{3}\right)\left(\frac{7}{2}\right)}$
Solution.
$Distributive\ property\ of\ multiplication\ over\ addition.$

(iii). $\boldsymbol{\pi + (-\pi) = 0}$
Solution.
$Additive\ Inverse.$

(iv). $\boldsymbol{\sqrt{3}.\sqrt{3}\ is\ a\ real\ number}.$
Solution.
$Closure\ law\ for\ multiplication.$

(v). $\boldsymbol{\left(-\frac{5}{8}\right)\left(-\frac{8}{5}\right) = 1}$
Solution.
$Multiplicative\ Inverse.$

Radicals and Radicands:

If n is a positive integer greater than 1 and a is a real number , then any real number x such that $x^n = a$ is called the nth root of a , $and\ in\ symbols\ is\ written\ as$

$$x = \sqrt[n]{a} \quad or\ x = (a)^{\frac{1}{n}}$$

And $\sqrt[n]{a}$ is called radical, the symbol $\sqrt{\ }$ is called the radical sign , n is called the index of the radical and the real number a under the radical sign is called the radicand or base.

Radical and Exponent Form:

$x = \sqrt[n]{a}\ is\ called\ redical\ form\ and\ a = x^{\frac{1}{n}}\ is\ called\ exponent\ form.$

Some Properties of Radicals:

(i). $\sqrt[n]{ab} = \sqrt[n]{a}\sqrt[n]{b}$

(ii). $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

(iii). $\sqrt[n]{\sqrt[m]{a}} = \sqrt[nm]{a}$

(iv). $\sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$

(v). $\sqrt[n]{a^n} = a$

Exercise # 2.3

Question.1. Write each radical expression in exponential notation and each exponential expression in radical notation. Do not simplify.

(i). $\sqrt[3]{-64} = (-64)^{\frac{1}{3}}$

Solution.

$$\sqrt[3]{-64} = (-64)^{\frac{1}{3}}$$

(ii). $2^{\frac{3}{5}}$

Solution.

$$2^{\frac{3}{5}} = \sqrt[5]{2^3}$$

(iii). $-7^{\frac{1}{3}}$

Solution.

$$-7^{\frac{1}{3}} = -\sqrt[3]{7}$$

(iv). $y^{-\frac{2}{3}}$

Solution.

$$y^{-\frac{2}{3}} = \sqrt[3]{y^{-2}}$$

Question.2. Tell whether the following statements are true or false?

(i). $5^{\frac{1}{5}} = \sqrt{5}$

Solution.

$$False\ because\ 5^{\frac{1}{5}} = \sqrt[5]{5}\ is\ true.$$

(ii). $2^{\frac{2}{3}} = \sqrt[3]{4}$

Solution.

$$True\ because\ 2^{\frac{2}{3}} = \sqrt[3]{2^2} = \sqrt[3]{4}\ is\ true.$$

(iii). $\sqrt{49} = \sqrt{7}$

Solution.

$$False\ because\ \sqrt{49} = \sqrt{7^2} = 7 is\ true.$$

(iv). $\sqrt[3]{x^{27}} = x^3$

Solution.

$$False\ because\ \sqrt[3]{x^{27}} = x^{\frac{27}{3}} = x^9\ is\ true.$$

Question.3. Simplify the following radical expressions.

(i). $\sqrt[3]{-125}$

Solution.

$$\sqrt[3]{-125} = \sqrt[3]{-5^3}$$
$$= (-5)^{3\times\frac{1}{3}}$$
$$= -5$$

Answer.

(ii). $\sqrt[4]{32}$

Solution.

$$\sqrt[4]{32} = \sqrt[4]{2^4 \times 2}$$
$$= (2^4 \times 2)^{\frac{1}{4}}$$
$$= (2^4)^{\frac{1}{4}} \times 2^{\frac{1}{4}}$$
$$= 2 \times \sqrt[4]{2}$$
$$= 2\sqrt[4]{2}$$

Answer.

(iii). $\sqrt[5]{\frac{3}{32}}$

Solution.

$$\sqrt[5]{\frac{3}{32}} = \left(\frac{3}{32}\right)^{\frac{1}{5}}$$
$$= \left(\frac{3}{2^5}\right)^{\frac{1}{5}}$$
$$= \frac{3^{\frac{1}{5}}}{2^{5\times\frac{1}{5}}}$$
$$= \frac{\sqrt[5]{3}}{2}$$

Answer.

(iv). $\sqrt[3]{\frac{-8}{27}}$

Solution.

$$\sqrt[3]{\frac{-8}{27}} = \left(\frac{-2^3}{3^3}\right)^{\frac{1}{3}}$$
$$= \frac{-2^{3\times\frac{1}{3}}}{3^{3\times\frac{1}{3}}}$$
$$= \frac{-2}{3}$$

Answer.

Base and Exponents:

In the exponential form

a^n $(read\ as\ a\ to\ the\ nth\ power)$ we call $''a''$ as the base and $''n''$ as the exponent or power.

Laws of Exponents:

If $a, b \in$

$R\ and\ m\ , n\ are\ positive\ integers, then$

(i). $a^m \,.\, a^n = a^{m+n}$

(ii). $(a^m)^n = a^{mn}$

(iii). $(ab)^n = a^n b^n$

(iv). $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

(v). $\frac{a^m}{a^n} = a^{m-n}$

(vi). $a^0 = 1\ , where\ a \neq 0$

(vii). $a^{-n} = \frac{1}{a^n}\ , where\ a \neq 0$

Exercise # 2.4

Question.1. Use laws of exponents to simplify

(i). $\frac{(243)^{-\frac{2}{3}}(32)^{-\frac{1}{5}}}{\sqrt{(196)^{-1}}}$

Solution.

$$\frac{(243)^{-\frac{2}{3}}(32)^{-\frac{1}{5}}}{\sqrt{(196)^{-1}}} = \frac{(3^5)^{-\frac{2}{3}}(2^5)^{-\frac{1}{5}}}{(14^2)^{-1\times\frac{1}{2}}}$$

$$= \frac{3^{-\frac{10}{3}}2^{-1}}{14^{-1}}$$

$$= \frac{3^{-\frac{10}{3}}2^{-1}}{(2\times 7)^{-1}}$$

$$= \frac{3^{-\frac{10}{3}}2^{-1}}{2^{-1}\times 7^{-1}}$$

$$= \frac{3^{-\frac{10}{3}}}{7^{-1}}$$

$$= \frac{7}{3^{\frac{9+1}{3}}}$$

$$= \frac{7}{3^{\frac{9}{3}+\frac{1}{3}}}$$

$$= \frac{7}{3^{\frac{9}{3}}\times 3^{\frac{1}{3}}}$$

$$= \frac{7}{3^3\times\sqrt[3]{3}}$$

$$= \frac{7}{27\sqrt[3]{3}}$$

Answer.

(ii). $(2x^5y^{-4})(-8x^{-3}y^2)$

Solution.

$$(2x^5y^{-4})(-8x^{-3}y^2) = (2)(-8)x^5.y^{-4}.x^{-3}y^2$$

$$= -16\,x^{5-3}.y^{-4+2}$$

$$= -16\,x^2.y^{-2}$$

$$= -\frac{16x^2}{y^2}$$

Answer.

(iii). $\left(\frac{x^{-2}y^{-1}z^{-4}}{x^4y^{-3}z^0}\right)^{-3}$

Solution.

$$\left(\frac{x^{-2}y^{-1}z^{-4}}{x^4y^{-3}z^0}\right)^{-3} = \left(\frac{y^{-1+3}}{x^{4+2}z^{0+4}}\right)^{-3}$$

$$= \left(\frac{y^2}{x^6z^4}\right)^{-3}$$

$$= \left(\frac{x^6z^4}{y^2}\right)^{3}$$

$$= \frac{x^{6\times 3}z^{4\times 3}}{y^{2\times 3}}$$

$$= \frac{x^{18}z^{12}}{y^6}$$

Answer.

(iv). $\frac{(81)^n.3^5-(3)^{4n-1}(243)}{(9)^{2n}.3^3}$

Solution.

$$\frac{(81)^n.3^5-(3)^{4n-1}(243)}{(9)^{2n}.3^3}$$

$$= \frac{(3^4)^n.3^5-(3)^{4n-1}(3)^5}{(3^2)^{2n}.3^3}$$

$$= \frac{(3)^{4n}.3^5-(3)^{4n-1}(3)^5}{(3)^{4n}.3^3}$$

$$= \frac{3^{4n+5}-3^{4n-1+5}}{3^{4n+3}}$$

$$= \frac{3^{4n+5}-3^{4n+4}}{3^{4n+3}}$$

$$= \frac{3^{4n+4}(3^1-1)}{3^{4n+3}}$$

$$= 3^{4n+4-4n-3}(2)$$

$$= 3^1(2)$$

$$= 6$$

Answer

Question.2. Show that

$$\left(\frac{x^a}{x^b}\right)^{a+b}\times\left(\frac{x^b}{x^c}\right)^{b+c}\times\left(\frac{x^c}{x^a}\right)^{c+a} = 1$$

Solution.

$$L.H.S = \left(\frac{x^a}{x^b}\right)^{a+b}\times\left(\frac{x^b}{x^c}\right)^{b+c}\times\left(\frac{x^c}{x^a}\right)^{c+a}$$

$$L.H.S = (x^{a-b})^{a+b}\times(x^{b-c})^{b+c}\times(x^{c-a})^{c+a}$$

$$L.H.S = x^{(a-b)(a+b)}\times x^{(b-c)(b+c)}\times x^{(c-a)(c+a)}$$

$$L.H.S = x^{a^2-b^2}\times x^{b^2-c^2}\times x^{c^2-a^2}$$

$$L.H.S = x^{a^2-b^2+b^2-c^2+c^2-a^2}$$

$$L.H.S = x^0 = 1$$

Hence Proved.

Question.3. Simplify

(i). $\frac{2^{\frac{1}{3}}(27)^{\frac{1}{3}}(60)^{\frac{1}{2}}}{(180)^{\frac{1}{2}}(4)^{-\frac{1}{3}}9^{\frac{1}{4}}}$

Solution.

$$\frac{2^{\frac{1}{3}}(27)^{\frac{1}{3}}(60)^{\frac{1}{2}}}{(180)^{\frac{1}{2}}(4)^{-\frac{1}{3}}9^{\frac{1}{4}}} = \frac{2^{\frac{1}{3}}(3^3)^{\frac{1}{3}}(2^2.3.5)^{\frac{1}{2}}}{(2^2.3^2.5)^{\frac{1}{2}}(2^2)^{-\frac{1}{3}}(3^2)^{\frac{1}{4}}}$$

$$= \frac{2^{\frac{1}{3}}3^1.2^{2\times\frac{1}{2}}.3^{\frac{1}{2}}.5^{\frac{1}{2}}}{2^{2\times\frac{1}{2}}.3^{2\times\frac{1}{2}}.5^{\frac{1}{2}}.2^{-\frac{2}{3}}3^{2\times\frac{1}{4}}}$$

$$= \frac{2^{\frac{1}{3}}3^1.2^1.3^{\frac{1}{2}}.5^{\frac{1}{2}}}{2^1.3^1.5^{\frac{1}{2}}.2^{-\frac{2}{3}}3^{\frac{1}{2}}}$$

$$= \frac{2^{\frac{1}{3}}}{2^{-\frac{2}{3}}}$$
$$= 2^{\frac{1}{3}+\frac{2}{3}}$$
$$= 2^{\frac{1+2}{3}}$$
$$= 2^{\frac{3}{3}}$$
$$=2$$
Answer.

(ii). $\sqrt{\frac{(216)^{\frac{2}{3}}(25)^{\frac{1}{2}}}{(0.04)^{-\frac{1}{2}}}}$

Solution.

$$\sqrt{\frac{(216)^{\frac{2}{3}}(25)^{\frac{1}{2}}}{(0.04)^{-\frac{1}{2}}}} = \sqrt{\frac{(6^3)^{\frac{2}{3}}(5^2)^{\frac{1}{2}}}{\left(\frac{4}{100}\right)^{-\frac{1}{2}}}}$$
$$= \sqrt{\frac{6^2\,.\,5^1}{\left(\frac{1}{25}\right)^{-\frac{1}{2}}}}$$
$$= \sqrt{\frac{6^2\,.\,5^1}{(25)^{\frac{1}{2}}}}$$
$$= \sqrt{\frac{6^2\,.5}{(5^2)^{\frac{1}{2}}}}$$
$$= \sqrt{\frac{6^2.5}{5}}$$
$$= \sqrt{36}$$
$$= 6$$
Answer.

(iii). $5^{2^3} \div (5^2)^3$

Solution.

$$5^{2^3} \div (5^2)^3 = \frac{5^8}{5^6}$$
$$= 5^{8-6}$$
$$= 5^2$$
$$= 25$$
Answer.

(iv). $(x^3)^2 \div x^{3^2}$

Solution.

$$(x^3)^2 \div x^{3^2} = \frac{x^6}{x^8}$$
$$= \frac{1}{x^{8-6}}$$
$$= \frac{1}{x^2}$$
Answer.

Complex Numbers:

The numbers of the form $x + iy$, where $x, y \in \Re$,are called **complex numbers,** here x is called **real part** and y is called **imaginary part** of the complex number.

Remarks:

1. Every real number is a complex number with 0 as its imaginary part.

Conjugate Complex Numbers:

$if\ Z = a + ib\ be\ a\ complex\ number\ then\ \bar{Z} = a - ib$ is the conjugate of the complex number $Z = a + ib$.

Remarks:

1. A real number is self-Conjugate.

Equality of Two Complex Numbers:

Two complex numbers $a + bi\ and\ c + di\ are\ said\ to\ be\ equal\ if\ a = c\ and\ b = d.$

That is

$$a + ib = c + id \quad \Rightarrow a = b\ and\ c = d.$$

Exercise # 2.5

Question.1. Evaluate

(i). $\boldsymbol{i^7}$

Solution.

$$\boldsymbol{i^7 = i^6.i}$$
$$\boldsymbol{= (i^2)^3.i}$$
$$\boldsymbol{= (-1)^3.i}$$
$$\boldsymbol{= (-1).i}$$
$$\boldsymbol{= -i}$$
Answer.

(ii). $\boldsymbol{i^{50}}$

Solution.

$$\boldsymbol{i^{50} = (i^2)^{25}}$$
$$\boldsymbol{= (-1)^{25}}$$
$$\boldsymbol{= -1}$$
Answer.

$(\boldsymbol{iii}).\boldsymbol{i^{12}}$

Solution.

$$\boldsymbol{i^{12} = (i^2)^6}$$
$$\boldsymbol{= (-1)^6}$$
$$\boldsymbol{= 1}$$
Answer.

(iv). $(\boldsymbol{-i})^8$

Solution.

$$\boldsymbol{(-i)^8 = i^8}$$
$$\boldsymbol{= (i^2)^4}$$
$$\boldsymbol{= (-1)^4}$$
$$\boldsymbol{= 1}$$
Answer.

(v). $(\boldsymbol{-i})^5$

Solution.

$$(-i)^5 = -i^5$$
$$= -i^4.i$$
$$= -(i^2)^2.i$$
$$= -(-1)^2.i$$
$$= -(1).i$$
$$= -i$$

Answer.

(vi). i^{27}

Solution.

$$i^{27} = i^{26}.i$$
$$= (i^2)^{13}.i$$
$$= (-1)^{13}.i$$
$$= (-1).i$$
$$= -1$$

Answer.

Question.2. Write the conjugate of the following numbers.

(i). $2+3i$

Solution.

$$Suppose\ \ Z = 2+3i$$
$$\bar{Z} = \overline{2+3i} = 2-3i$$

Answer.

(ii). $3-5i$

Solution.

$$Suppose\ \ Z = 3-5i$$
$$\bar{Z} = \overline{3-5i} = 3+5i$$

Answer.

(iii). $-i$

Solution.

$$Suppose\ \ Z = -i$$
$$\bar{Z} = \overline{-i} = +i$$

Answer.

(iv). $-3+4i$

Solution.

$$Suppose\ \ Z = -3+4i$$
$$\bar{Z} = \overline{-3+4i} = -3-4i$$

Answer.

(v). $-4-i$

Solution.

$$Suppose\ \ Z = -4-i$$
$$\bar{Z} = \overline{-4-i} = -4+i$$

Answer.

Question.3. Write the real and imaginary part of the following numbers.

(i). $1+i$

Solution.

$$Suppose\ \ Z = 1+i$$
$$Re(Z) = 1\ \ ,\qquad Im(Z) = 1$$

Answer.

(ii). $-1+2i$

Solution.

$$Suppose\ \ Z = -1+2i$$
$$Re(Z) = -1\ \ ,\qquad Im(Z) = 2$$

Answer.

(iii). $-3i+2$

Solution.

$$Suppose\ \ Z = -3i+2 = 2-3i$$
$$Re(Z) = 2\ \ ,\qquad Im(Z) = -3$$

Answer.

(iv). $-2-2i$

Solution.

$$Suppose\ \ Z = -2i-2 = -2-2i$$
$$Re(Z) = -2\ \ ,\qquad Im(Z) = -2$$

Answer.

(v). $-3i$

Solution.

$$Suppose\ \ Z = -3i = 0-3i$$
$$Re(Z) = 0\ \ ,\qquad Im(Z) = -3$$

Answer.

(vi). $2+0i$

Solution.

$$Suppose\ \ Z = 2+0i$$
$$Re(Z) = 2\ \ ,\qquad Im(Z) = 0$$

Answer.

Question.4. Find the value of x and y if

$$x+iy+1 = 4-3i$$

Solution.

Given that

$$x+1+iy = 4-3i$$

Separating real and imaginary parts

$$x+1 = 4\ \ ,\quad y = -3$$
$$x = 4-1\ \ ,\quad y = -3$$
$$x = 3\ \ ,\quad y = -3$$

Answer.

Operations on Complex Numbers:

The symbols *a,b,c,d,k*, where used, represent real numbers

Addition of Two Complex Numbers:

$$(a+ib)+(c+id) = (a+b)+i(c+d).$$

Scalar Multiplication:

$$k(a+ib) = ka+ikb.$$

Subtraction of Two Complex Numbers:

$$(a+ib)-(c+id) = (a-b)+i(c-d).$$

Multiplication of Two Complex Numbers:

$$(a+ib)(c+id) = (ac-bd)+i(ad+bc).$$

Division of two Complex Numbers:

$$\frac{(a+ib)}{(c+id)} = \frac{ac-bd}{c^2+d^2}+i\frac{bc-ad}{c^2+d^2}$$

Exercise # 2.6

Question.1. Identify the following statements as true or false.

(i). $\sqrt{-3}\times\sqrt{-3}=3$

Solution.

$$False\ because\ \sqrt{-3}\times\sqrt{-3}=\sqrt{3}i\times\sqrt{3}i$$
$$=\left(\sqrt{3}\right)^2 i^2=-3$$

(ii). $i^{73}=-i$

Solution.

$$False\ because\ i^{73}=i^{72}.i=(i^2)^{36}.i$$
$$=(-1)^{36}.i=i$$

(iii). $i^{10}=-1$

Solution.

$$True\ because\ i^{10}=(i^2)^5=(-1)^5=-1$$

(iv). $Complex\ conjugate\ of\ (-6i+i^2)\ is\ (-1+6i)$

Solution.

$$True\ because\ \overline{-6i+i^2}=\overline{-6i-1}=-1+6i$$

(v). Difference of a complex number $z=a+bi\ and\ its\ conjugate\ is\ a\ real\ number.$

Solution.

$$False\ because\ Z-\overline{Z}=(a+bi)-(a-bi)$$
$$=a+bi-a+bi=2bi$$

(vi). If $(a-1)-(b+3)i=5+8i\ then\ a=6\ and\ \ b=-11.$

Solution.

True because Comparing real and imaginary parts in given equation

$$a-1=5\quad,\quad -(b+3)=8$$
$$a=5+1\quad,\quad b+3=-8$$
$$a=6\quad,\quad b=-8-3$$
$$a=6\quad,\quad b=-11$$

(vii) Product of a complex number and its conjugate is always a non-negative real number.

Solution.

$$\boldsymbol{True\ because\ for\ a\ complex\ numer\ Z=a+bi}$$
$$\boldsymbol{Z.\overline{Z}=(a+bi).(a-bi)=a^2-(bi)^2}$$
$$\boldsymbol{=a^2+b^2}$$

Is a real number.

Question.2. Express each complex number in the standard form $a+bi$ where 'a' and 'b' are real numbers.

(i). $(2+3i)+(7-2i)$

Solution.

$$(2+3i)+(7-2i)=2+3i+7-2i$$
$$=\boldsymbol{9+i}$$

Answer.

(ii). $\boldsymbol{2(5+4i)-3(7+4i)}$

Solution.

$$\boldsymbol{2(5+4i)-3(7+4i)=10+8i-21-12i}$$
$$\boldsymbol{=-11-3i}$$

Answer.

(iii). $\boldsymbol{-1(-3+5i)-(4+9i)}$

Solution.

$$\boldsymbol{-1(-3+5i)-(4+9i)=3-5i-4-9i}$$
$$\boldsymbol{=-1-14i}$$

Answer.

(iv). $\boldsymbol{2i^2+6i^3+3i^{16}-6i^{19}+4i^{25}}$

Solution.

$$\boldsymbol{2i^2+6i^3+3i^{16}-6i^{19}+4i^{25}}$$
$$\boldsymbol{=2(-1)+6i^2i+3i^{16}-6i^{18}i+4i^{24}i}$$
$$\boldsymbol{=2(-1)+6(-1)i+3(i^2)^8-6(i^2)^9i+4(i^2)^{12}i}$$
$$\boldsymbol{=-2-6i+3(-1)^8-6(-1)^9i+4(-1)^{12}i}$$
$$\boldsymbol{=-2-6i+3(1)-6(-1)i+4(1)i}$$
$$\boldsymbol{=-2-6i+3+6i+4i}$$
$$\boldsymbol{=1+4i}$$

Question.3. Simplify and write your answer in the form $a+bi$.

(i). $\boldsymbol{(-7+3i)(-3+2i)}$

Solution.

$$\boldsymbol{(-7+3i)(-3+2i)=21-14i-9i+6i^2}$$
$$\boldsymbol{=21-14i-9i-6}$$
$$\boldsymbol{=15-23i}$$

Answer.

(ii). $\boldsymbol{(2-\sqrt{-4})(3-\sqrt{-4})}$

Solution.

$$\boldsymbol{(2-\sqrt{-4})(3-\sqrt{-4})=(2-2i)(3-2i)}$$
$$\boldsymbol{=2(3-2i)-2i(3-2i)}$$
$$\boldsymbol{=6-4i-6i+4i^2}$$
$$\boldsymbol{=6-10i-4}$$
$$\boldsymbol{=2-10i}$$

Answer.

(iii). $\boldsymbol{\left(\sqrt{5}-3i\right)^2}$

Solution.

$$\boldsymbol{\left(\sqrt{5}-3i\right)^2=\left(\sqrt{5}\right)^2+(3i)^2-2(\sqrt{5})(3i)}$$
$$\boldsymbol{=5+9i^2-6\sqrt{5}i}$$
$$\boldsymbol{=5-9-6\sqrt{5}i}$$
$$\boldsymbol{=-4-6\sqrt{5}i}$$

Answer.

(iv). $\boldsymbol{(2-3i)(\overline{3-2i})}$

Solution.

$$\boldsymbol{(2-3i)(\overline{3-2i})=(2-3i)(3+2i)}$$
$$\boldsymbol{=2(3+2i)-3i(3+2i)}$$
$$\boldsymbol{=6+4i-9i-6i^2}$$
$$\boldsymbol{=6-5i+6}$$

$$= 12 - 5i$$

Answer.

Question.4. Simplify and write your answer in the form of $a + bi$.

(i). $-\frac{2}{1+i}$

Solution.

$$-\frac{2}{1+i} = \frac{-2}{1+i} \times \frac{1-i}{1-i}$$
$$= \frac{-2+2i}{1^2 - i^2}$$
$$= \frac{-2+2i}{1+1}$$
$$= \frac{-2+2i}{2}$$
$$= -\frac{2}{2} + \frac{2i}{2}$$
$$= -1 + i$$

Answer.

(ii). $\frac{2+3i}{4-i}$

Solution.

$$\frac{2+3i}{4-i} = \frac{2+3i}{4-i} \times \frac{4+i}{4+i}$$
$$= \frac{2(4+i) + 3i(4+i)}{4^2 - i^2}$$
$$= \frac{8 + 2i + 12i + 3i^2}{16+1}$$
$$= \frac{8 + 14i - 3}{17}$$
$$= \frac{4 + 14i}{17}$$
$$= \frac{4}{17} + \frac{14}{17}i$$

Answer.

(iii). $\frac{9-7i}{3+i}$

Solution.

$$\frac{9-7i}{3+i} = \frac{9-7i}{3+i} \times \frac{3-i}{3-i}$$
$$= \frac{9(3-i) - 7i(3-i)}{3^2 - i^2}$$
$$= \frac{27 - 9i - 21i + 7i^2}{9+1}$$
$$= \frac{27 - 30i - 7}{10}$$
$$= \frac{20 - 30i}{10}$$
$$= \frac{20}{10} - \frac{30}{10}i$$
$$= 2 - 3i$$

Answer.

(iv). $\frac{2-6i}{3+i} - \frac{4+i}{3+i}$

Solution.

$$\frac{2-6i}{3+i} - \frac{4+i}{3+i} = \frac{2-6i}{3+i} \times \frac{3-i}{3-i} - \frac{4+i}{3+i} \times \frac{3-i}{3-i}$$
$$= \frac{2(3-i) - 6i(3-i)}{3^2 - i^2} - \frac{4(3-i) + i(3-i)}{3^2 - i^2}$$
$$= \frac{6 - 2i - 18i + 6i^2}{9+1}$$
$$- \frac{12 - 4i + 3i - i^2}{9+1}$$
$$= \frac{6 - 20i - 6}{10} - \frac{12 - i + 1}{10}$$
$$= \frac{-20i}{10} - \frac{13 - i}{10}$$
$$= \frac{-20i - 13 + i}{10}$$
$$= \frac{-20i - 13 + i}{10}$$
$$= \frac{-13 - 19i}{10}$$
$$= -\frac{13}{10} - \frac{19}{10}i$$

Answer.

(v). $\left(\frac{1+i}{1-i}\right)^2$

Solution.

$$\left(\frac{1+i}{1-i}\right)^2 = \left(\frac{1+i}{1-i} \times \frac{1+i}{1+i}\right)^2$$
$$= \left(\frac{1(1+i) + i(1+i)}{1^2 - i^2}\right)^2$$
$$= \left(\frac{1 + i + i + i^2}{1+1}\right)^2$$
$$= \left(\frac{1 + 2i - 1}{2}\right)^2$$
$$= \left(\frac{2i}{2}\right)^2$$
$$= i^2$$
$$= -1$$

Answer.

(vi). $\frac{1}{(2+3i)(1-i)}$

Solution.

$$\frac{1}{(2+3i)(1-i)} = \frac{1}{2(1-i) + 3i(1-i)}$$
$$= \frac{1}{2 - 2i + 3i - 3i^2}$$
$$= \frac{1}{2 + i + 3}$$
$$= \frac{1}{5+i}$$
$$= \frac{1}{5+i} \times \frac{5-i}{5-i}$$
$$= \frac{5-i}{5^2 - (i)^2}$$

$$= \frac{5-i}{25+1}$$
$$= \frac{5-i}{26}$$
$$= \frac{5}{26} - \frac{i}{26}$$

Answer.

Question.5.

Calculate (a) $\overline{Z}$ (b) $Z+\overline{Z}$ (c) $Z-\overline{Z}$ (d) $Z\overline{Z}$ for each of the following

(i). $Z = -i$

Solution.

(a). $\overline{Z} = \overline{-i} = i$

(b). $Z + \overline{Z} = -i + i = 0$

(c). $Z - \overline{Z} = (-i) - (i) = -i - i = -2i$

(d). $Z\overline{Z} = (-i)(i) = -i^2 = 1$

(ii). $Z = 2 + i$

Solution.

(a). $\overline{Z} = \overline{2+i} = 2 - i$

(b). $Z + \overline{Z} = 2 + i + 2 - i = 4$

(c). $Z - \overline{Z} = (2+i) - (2-i) = 2 + i - 2 + i = 2i$

(d). $Z\overline{Z} = (2+i)(2-i) = 2^2 - i^2 = 4 + 1 = 5$

(iii). $Z = \frac{1+i}{1-i}$

Solution.

$$Z = \frac{1+i}{1-i}$$
$$Z = \frac{1+i}{1-i} \times \frac{1+i}{1+i}$$
$$Z = \frac{1+i+i+i^2}{1+1}$$
$$Z = \frac{1+2i-1}{2}$$
$$Z = \frac{2i}{2}$$
$$Z = i$$

(a). $\overline{Z} = \overline{i} = -i$

(b). $Z + \overline{Z} = i - i = 0$

(c). $Z - \overline{Z} = (i) - (-i) = i + i = 2i$

(d). $Z\overline{Z} = (i)(-i) = -i^2 = 1$

(iv). $Z = \frac{4-3i}{2+4i}$

Solution.

$$Z = \frac{4-3i}{2+4i}$$
$$Z = \frac{4-3i}{2+4i} \times \frac{2-4i}{2-4i}$$
$$Z = \frac{8-16i-6i+12i^2}{2^2-(4i)^2}$$
$$Z = \frac{8-22i-12}{4-16i^2}$$
$$Z = \frac{-4-22i}{4+16}$$
$$Z = \frac{-4-22i}{20}$$
$$Z = -\frac{4}{20} - \frac{22}{20}i$$
$$Z = -\frac{1}{5} - \frac{11}{10}i$$

(a). $\overline{Z} = \overline{-\frac{1}{5} - \frac{11}{10}i} = -\frac{1}{5} + \frac{11}{10}i$

(b). $Z + \overline{Z} = -\frac{1}{5} - \frac{11}{10}i + -\frac{1}{5} + \frac{11}{10}i$

$$Z + \overline{Z} = -\frac{1}{5} - \frac{1}{5} = \frac{-1-1}{5} = -\frac{2}{5} = -\frac{2}{5}$$
$$Z + \overline{Z} = -\frac{2}{5}$$

(c). $Z - \overline{Z} = \left(-\frac{1}{5} - \frac{11}{10}i\right) - \left(-\frac{1}{5} + \frac{11}{10}i\right)$

$$Z - \overline{Z} = -\frac{1}{5} - \frac{11}{10}i + \frac{1}{5} - \frac{11}{10}i$$
$$Z - \overline{Z} = -\frac{11}{10}i - \frac{11}{10}i$$
$$Z - \overline{Z} = \frac{-11-11}{10}i$$
$$Z - \overline{Z} = -\frac{22}{10}i$$
$$Z - \overline{Z} = -\frac{11}{5}i$$

(d). $Z\overline{Z} = \left(-\frac{1}{5} - \frac{11}{10}i\right)\left(-\frac{1}{5} + \frac{11}{10}i\right)$

$$Z\overline{Z} = \left(-\frac{1}{5}\right)^2 - \left(\frac{11}{10}i\right)^2$$
$$Z\overline{Z} = \frac{1}{25} - \frac{121}{100}i^2$$
$$Z\overline{Z} = \frac{1}{25} + \frac{121}{100}$$
$$Z\overline{Z} = \frac{4+121}{100}$$
$$Z\overline{Z} = \frac{125}{100}$$
$$Z\overline{Z} = \frac{5}{4}$$

Answer.

Question.6. If $z = 2 + 3i$, $w = 5 - 4i$, *show that*

(i). $\overline{z + w} = \overline{z} + \overline{w}$

Solution.

$$L.H.S = \overline{z + w}$$
$$L.H.S = \overline{2 + 3\iota + 5 - 4\iota}$$
$$L.H.S = \overline{8 - \iota}$$
$$L.H.S = 8 + i - - - (1)$$
$$R.H.S = \overline{z} + \overline{w}$$
$$R.H.S = \overline{2 + 3\iota} + \overline{5 - 4\iota}$$
$$R.H.S = 2 - 3i + 5 + 4i$$

$$R.H.S = 8 + i --- (2)$$

From (1) and (2) , we have

$$L.H.S = R.H.S$$

Hence Proved.

(ii). $\overline{z-w} = \bar{z} - \bar{w}$

Solution.

$$L.H.S = \overline{z-w}$$
$$L.H.S = \overline{(2+3\imath)-(5-4\imath)}$$
$$L.H.S = \overline{2+3\imath-5+4\imath}$$
$$L.H.S = \overline{-3+7\imath}$$
$$L.H.S = -3-7i --- (1)$$
$$R.H.S = \bar{z} - \bar{w}$$
$$R.H.S = (\overline{2+3\imath}) - (\overline{5-4\imath})$$
$$R.H.S = (2-3i)-(5+4i)$$
$$R.H.S = 2-3i-5-4i$$
$$R.H.S = -3-7i --- (2)$$

From (1) and (2) , we have

$$L.H.S = R.H.S$$

Hence Proved.

(iii). $\overline{zw} = \bar{z}\,\bar{w}$

Solution.

$$L.H.S = \overline{zw}$$
$$L.H.S = \overline{(2+3\imath)(5-4\imath)}$$
$$L.H.S = \overline{10-8\imath+15\imath-12\imath^2}$$
$$L.H.S = \overline{10+7\imath+12}$$
$$L.H.S = \overline{22+7\imath}$$
$$L.H.S = 22-7i --- (1)$$
$$R.H.S = \bar{z}\bar{w}$$
$$R.H.S = (\overline{2+3\imath})(\overline{5-4\imath})$$
$$R.H.S = (2-3i)(5+4i)$$
$$R.H.S = 10+8i-15i-12i^2$$
$$R.H.S = 10-7i+12$$
$$R.H.S = 22-7i --- (2)$$

From (1) and (2) , we have

$$L.H.S = R.H.S$$

Hence Proved.

(iv). $\overline{\left(\frac{z}{w}\right)} = \frac{\bar{z}}{\bar{w}}$

Solution.

$$L.H.S = \overline{\left(\frac{z}{w}\right)}$$
$$L.H.S = \overline{\left(\frac{2+3\imath}{5-4\imath}\right)}$$
$$L.H.S = \overline{\left(\frac{2+3\imath}{5-4\imath}\times\frac{5+4\imath}{5+4\imath}\right)}$$
$$L.H.S = \overline{\left(\frac{10+8\imath+15\imath+12\imath^2}{5^2-(4\imath)^2}\right)}$$
$$L.H.S = \overline{\left(\frac{10+23\imath-12}{25-16\imath^2}\right)}$$
$$L.H.S = \overline{\left(\frac{-2+23\imath}{25+16}\right)}$$
$$L.H.S = \overline{\left(\frac{-2+23\imath}{41}\right)}$$
$$L.H.S = \overline{\left(-\frac{2}{41}+\frac{23}{41}\imath\right)}$$
$$L.H.S = -\frac{2}{41}-\frac{23}{41}i --- (1)$$
$$R.H.S = \frac{\bar{z}}{\bar{w}}$$
$$R.H.S = \frac{(\overline{2+3\imath})}{\overline{5-4\imath}}$$
$$R.H.S = \frac{2-3i}{5+4i}$$
$$R.H.S = \frac{2-3i}{5+4i}\times\frac{5-4i}{5-4i}$$
$$R.H.S = \frac{10-8i-15i+12i^2}{5^2-(4i)^2}$$
$$R.H.S = \frac{10-23i-12}{25-16i^2}$$
$$R.H.S = \frac{-2-23i}{25+16}$$
$$R.H.S = \frac{-2-23i}{41}$$
$$R.H.S = -\frac{2}{41}-\frac{23}{41}i --- (2)$$

From (1) and (2) , we have

$$L.H.S = R.H.S$$

Hence Proved.

(v). $\frac{1}{2}(z+\bar{z})$ *is a real part of z.*

Solution.

$$\frac{1}{2}(z+\bar{z}) = \frac{1}{2}\left(2+3i+\overline{2+3\imath}\right)$$
$$= \frac{1}{2}(2+3i+2-3i)$$
$$= \frac{1}{2}(4)$$
$$= 2 \text{ which is real part of } z.$$

Hence Proved.

(vi). $\frac{1}{2i}(z-\bar{z})$ *is a imaginary part of z.*

Solution.

$$\frac{1}{2i}(z-\bar{z}) = \frac{1}{2i}\left((2+3i)+(\overline{2+3\imath})\right)$$
$$= \frac{1}{2i}\left((2+3i)-(2-3i)\right)$$
$$= \frac{1}{2i}(2+3i-2+3i)$$
$$= \frac{1}{2i}(6i)$$
$$= 3 \text{ which is imaginary part of } z.$$

Hence Proved.

Question.7. Solve the following equations for real x and y.

(i). $(2-3i)(x+iy) = 4+i$

Solution. Given that

$$(2-3i)(x+iy)=4+i$$
$$2(x+iy)-3i(x+iy)=4+i$$
$$2x+2iy-3ix-3i^2y=4+i$$
$$2x+2iy-3ix+3y=4+i$$
$$2x+3y+(2y-3x)i=4+i$$

Comparing real and imaginary parts, we have

$$2x+3y=4---(i)\ ,\quad 2y-3x=1---(ii)$$

$3\times(i)+2\times(ii), we\ have$

$$3(2x+3y)+2(2y-3x)=3(4)+2(1)$$
$$6x+9y+4y-6x=12+2$$
$$13y=14$$
$$y=\frac{14}{13}$$

Using value of $y\ in\ equation\ (i), we\ have$

$$2x+3\left(\frac{14}{13}\right)=4$$
$$2x+\frac{42}{13}=4$$
$$2x=4-\frac{42}{13}$$
$$2x=\frac{4\times13-42}{13}$$
$$2x=\frac{52-42}{13}$$
$$2x=\frac{10}{13}$$
$$x=\frac{10}{13\times2}$$
$$x=\frac{5}{13}$$

Hence required $x=\frac{5}{13}\ and\ y=\frac{14}{13}$.

(ii). $(3-2i)(x+iy)=2(x-2yi)+2i-1$

Solution. Given that

$$(3-2i)(x+iy)=2(x-2yi)+2i-1$$
$$3(x+iy)-2i(x+iy)=2x-4yi+2i-1$$
$$3x+3iy-2ix-2i^2y=2x-1+2i-4yi$$
$$3x+3iy-2ix+2y=2x-1+(2-4y)i$$
$$3x+2y+(3y-2x)i=2x-1+(2-4y)i$$

Comparing real and imaginary parts, we have

$$3x+2y=2x-1\ ,(3y-2x)=2-4y$$
$$3x-2x+2y=-1\ ,-2x+3y+4y=2$$
$$x+2y=-1---(i)\ ,-2x+7y=2---(ii)$$

$2\times(i)+(ii), we\ have$

$$2(x+2y)+(-2x+7y)=2(-1)+2$$
$$2x+4y-2x+7y=-2+2$$
$$11y=0$$
$$y=0$$

Using value of $y\ in\ equation\ (i), we\ have$

$$x+2(0)=-1$$
$$x=-1$$

Hence required $x=-1\ and\ y=0$.

(iii). $(3+4i)^2-2(x-iy)=x+yi$

Solution. Given that

$$(3+4i)^2-2(x-iy)=x+yi$$
$$(3)^2+(4i)^2+2(3)(4i)-2x+2iy=x+yi$$
$$9+16i^2+12i-2x+2iy=x+yi$$
$$9-16+12i-2x+2iy=x+yi$$
$$-7-2x+(12+2y)i=x+yi$$

Comparing real and imaginary parts, we have

$$-7-2x=x\ ,12+2y=y$$
$$x+2x=-7=\ ,2y-y=12$$
$$3x=-7\ ,y=12$$
$$x=-\frac{7}{3}\ ,y=12$$

Hence required $x=-\frac{7}{3}\ and\ y=12$.